

FCC Method of Moments

Licensing AM
Directional Antenna
Systems

The Task

- Comply with Construction Permit
 - Certify correct antenna location
 - Certify compliant pattern shape
 - Certify compliant pattern size
- Prepare FCC “302”
- Propose Verifiable Operating Parameters

Classic AM “Proof”

- Construction Permit Requires
 - Un-attenuated Inverse Field
- Field Measurements Provide
 - Attenuated Inverse Field
- Need to Determine Ground Conductivity
 - Build and measure a reference antenna

Classic AM “Proof”

- Establish Pattern Shape and Size
 - Measure directional antenna fields
 - Apply ground conductivity correction
 - Compare results to construction permit
 - Adjust phasor controls
 - Repeat as necessary...
 - Perhaps hundreds of field measurements
- A Time (and Money) Consuming Process

Classic “AM “ Proof

■ Pattern Shape

- A function of individual tower contributions
- Antenna monitor and sampling system

■ Antenna Monitor Parameters

- A “snapshot” on the last day of adjustment
- For reference, absolute value means little
- Filed with FCC “302” – License Parameters

Classic AM “Proof”

- Long Term Maintenance
 - “Periodic” License Parameter Readings
 - “Periodic” Monitor Point readings
- Future Antenna System Changes
 - More field measurements
 - Reference original ground conductivity
 - More repeat as necessary...

Could This
Cumbersome Process
be Improved?

FCC Method of Moments

- PRM 08-228 Second Report and Order
 - Originally FCC MM 93-177
 - Originally Proposed in 1991
 - Hatfield and Dawson
 - duTreil, Lundin & Rackley
 - Lahm, Suffa & Cavell
 - Moffet, Larson & Johnson
 - Silliman & Silliman
 - Available for Use in Filings Early in 2009

“Method of Moments”

- Applicable to Engineering Problems
 - Acoustics, Fluids, Mechanics
 - Electromagnetics

- Also Known as:
 - Boundary Element Method
 - Finite Element Method
 - Finite Difference Method

“Method of Moments”

■ From “Wikipedia”:

- a numerical computational method of solving linear partial differential equations which have been formulated as integral equations (i.e. in *boundary integral* form).
- In electromagnetics, the more traditional term "method of moments" is often, though not always, synonymous with "boundary element method"

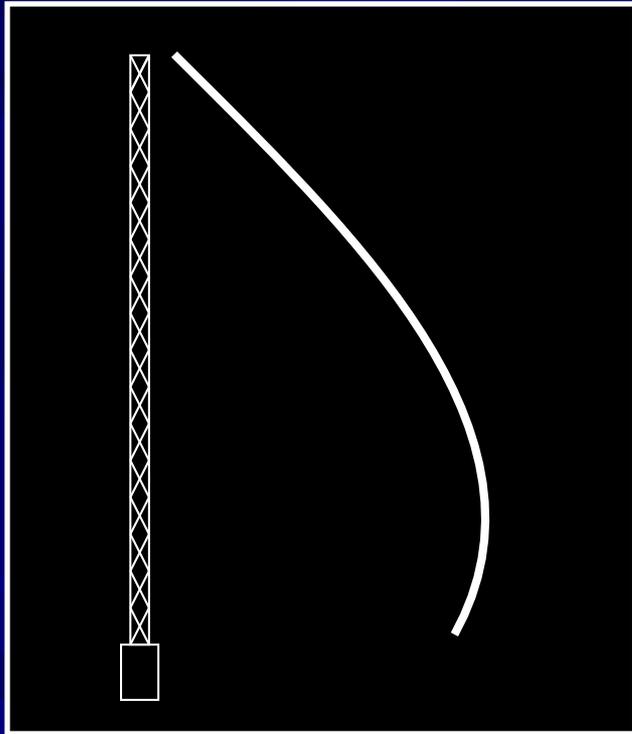
“Method of Moments”

- In Plain English:
 - Take a large complicated problem
 - Cut it up into small, simpler pieces
 - Solve the pieces (perhaps simultaneously)
 - Recombine the solutions

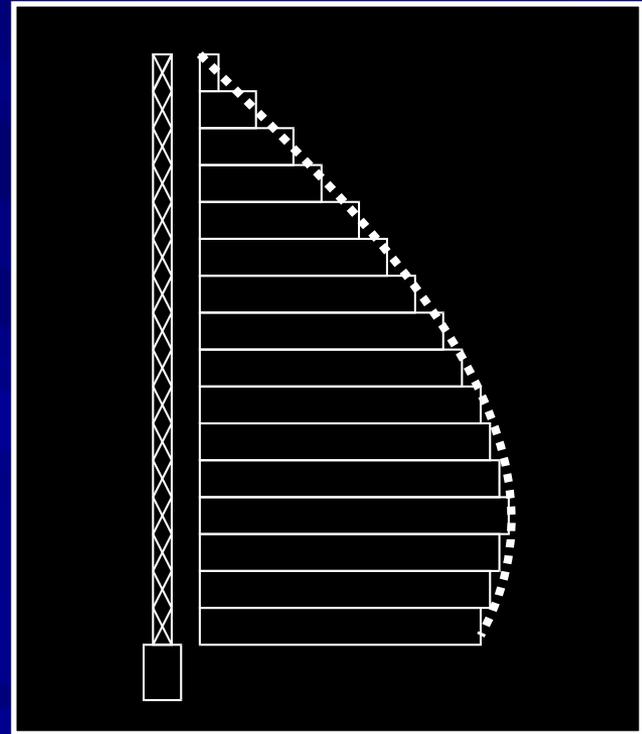
- The Numerical Electromagnetics Code
 - Lawrence Livermore National Laboratory

“Method of Moments”

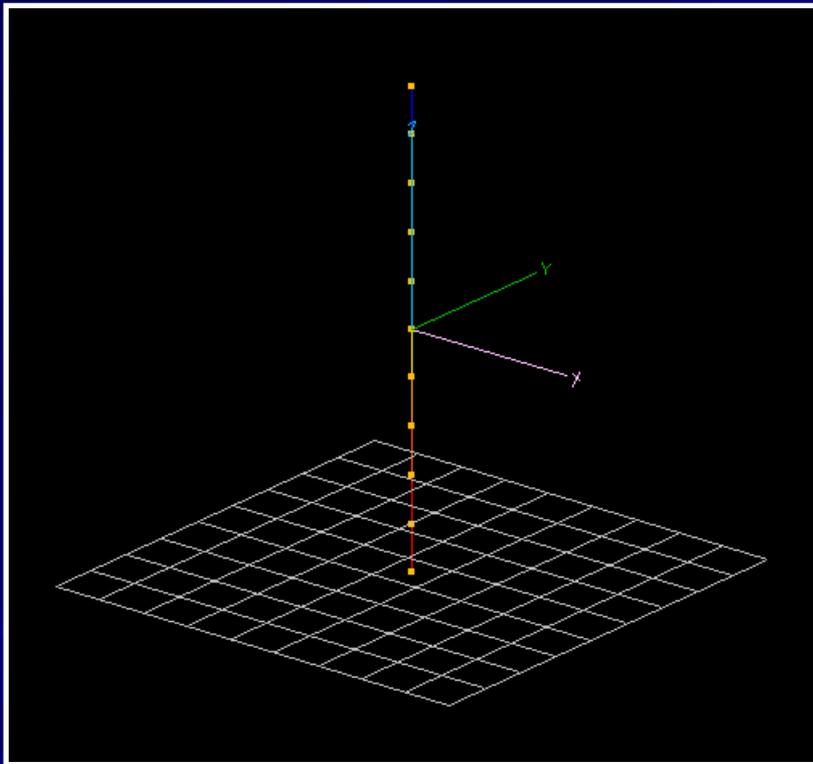
Continuous Current



Segmented Current



“Method of Moments”



NEC Modeled Antenna

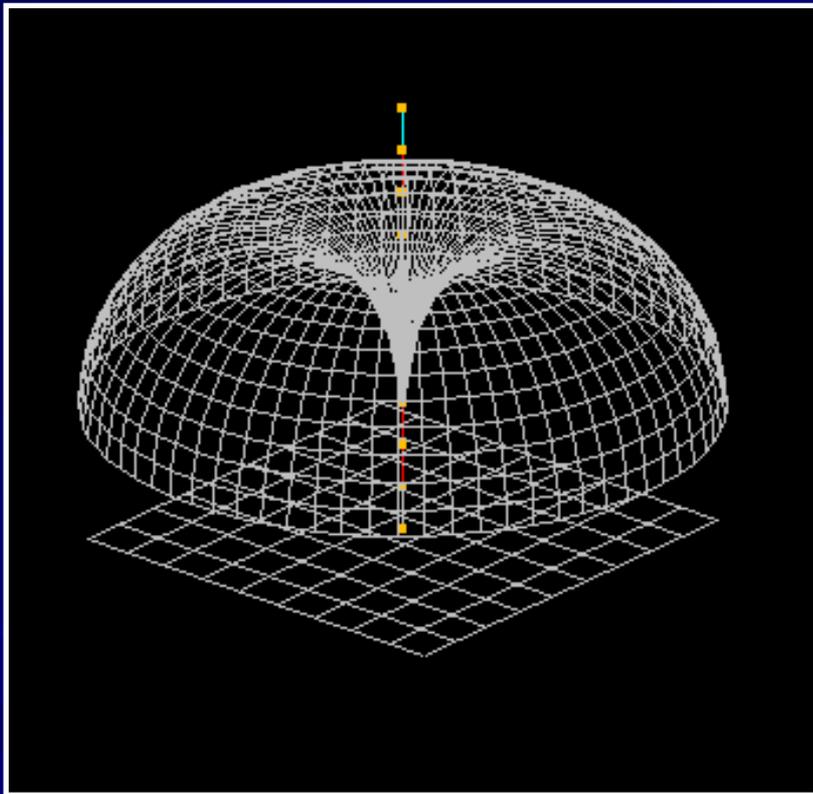
$\frac{1}{4}$ Wave Vertical

“Perfect” ground

10 Segments

Color indicates currents

“Method of Moments”



NEC Modeled Antenna

“Wire Frame” patterns

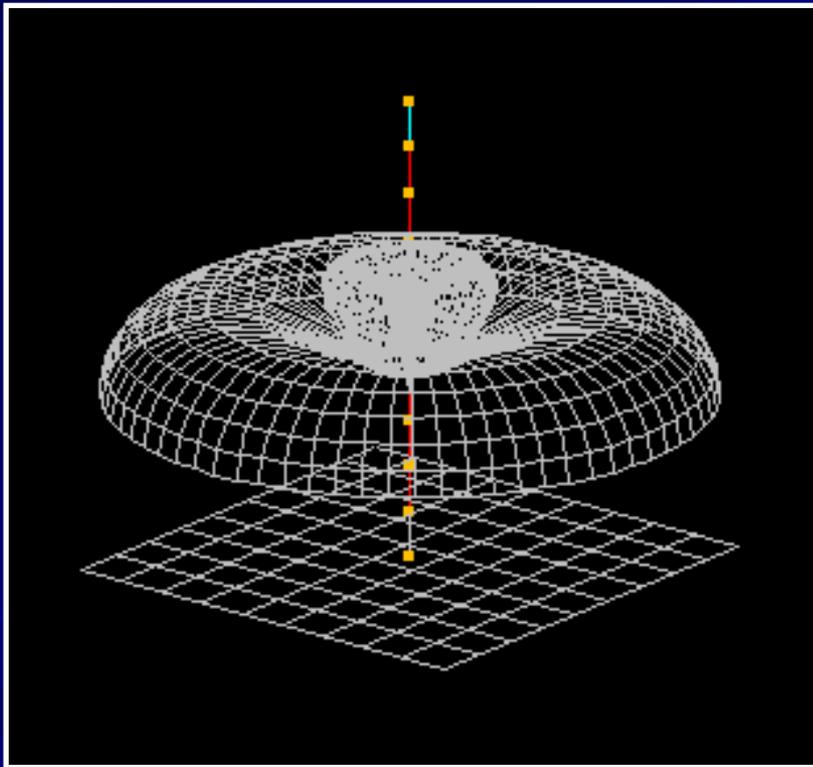
Polar plots

Antenna gain

Drive Point Impedance

about $36 j0$ ohms

“Method of Moments”



NEC Modeled Antenna

5/8 Wave Vertical
Note Skywave Lobes

Drive Point Impedance
about $205 - j613$ ohms

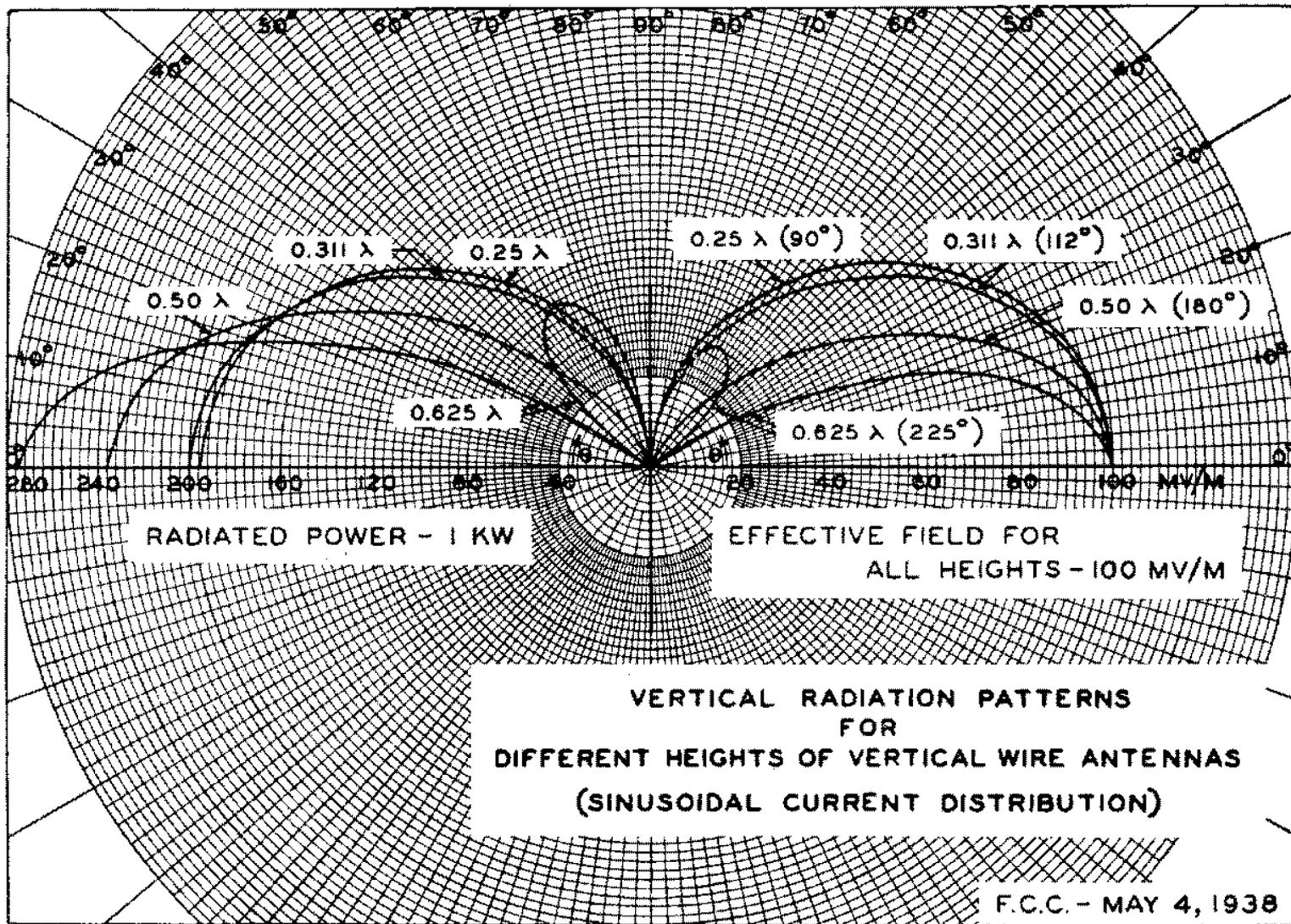
Back to “The Task”

- Consider a Non-Directional “302”
 - Determine Location: USGS Map
 - Determine Pattern Shape
 - Defined by geometry in horizontal plane
 - Defined by formula in vertical plane

$$f(\theta) = \frac{\cos(G \sin \theta) - \cos G}{(1 - \cos G) \cos \theta}$$

(FCC Rules 73.160)

$$f(\theta) = \frac{\cos(G \sin \theta) - \cos G}{(1 - \cos G) \cos \theta}$$



NDA Pattern Size

- Un-attenuated Field (specified in mV/m)
 - Difficult to directly measure
- Proportional to Power (in Watts)
 - Difficult to directly measure
- Proportional to Base Current (in Amperes)
 - Easily measured if base impedance is known

NDA Base Impedance

- Determined by:
 - Tower height, cross-section, material
 - Base Insulator, lighting chokes
 - Isocouplers, stray reactance, etc.
- Measure Impedance
 - Not expected to change under power
 - Required base current can be calculated

NDA FCC “302”

- Location – coordinates from USGS Map
- Pattern Shape – geometry and formula
- Pattern Size – calculated base current

We have obtained the required technical information to prepare the license application without ever actually turning on the transmitter.

Back to “The Task”

- FCC Method of Moments for DA's
 - Modeling the directional array
 - Verifying the mathematical model
 - Establishing operating parameters
 - Sampling system requirements
 - Long term maintenance
 - Additional FCC requirements

Modeling

■ NEC Model

- Mathematical description of array
- Excite each tower, “float” all others
- Create a tower array impedance matrix

■ Physical Array

- Measure each tower, “float” all others
- Characterize any extra “stuff”

■ “Massage” the Model

Example

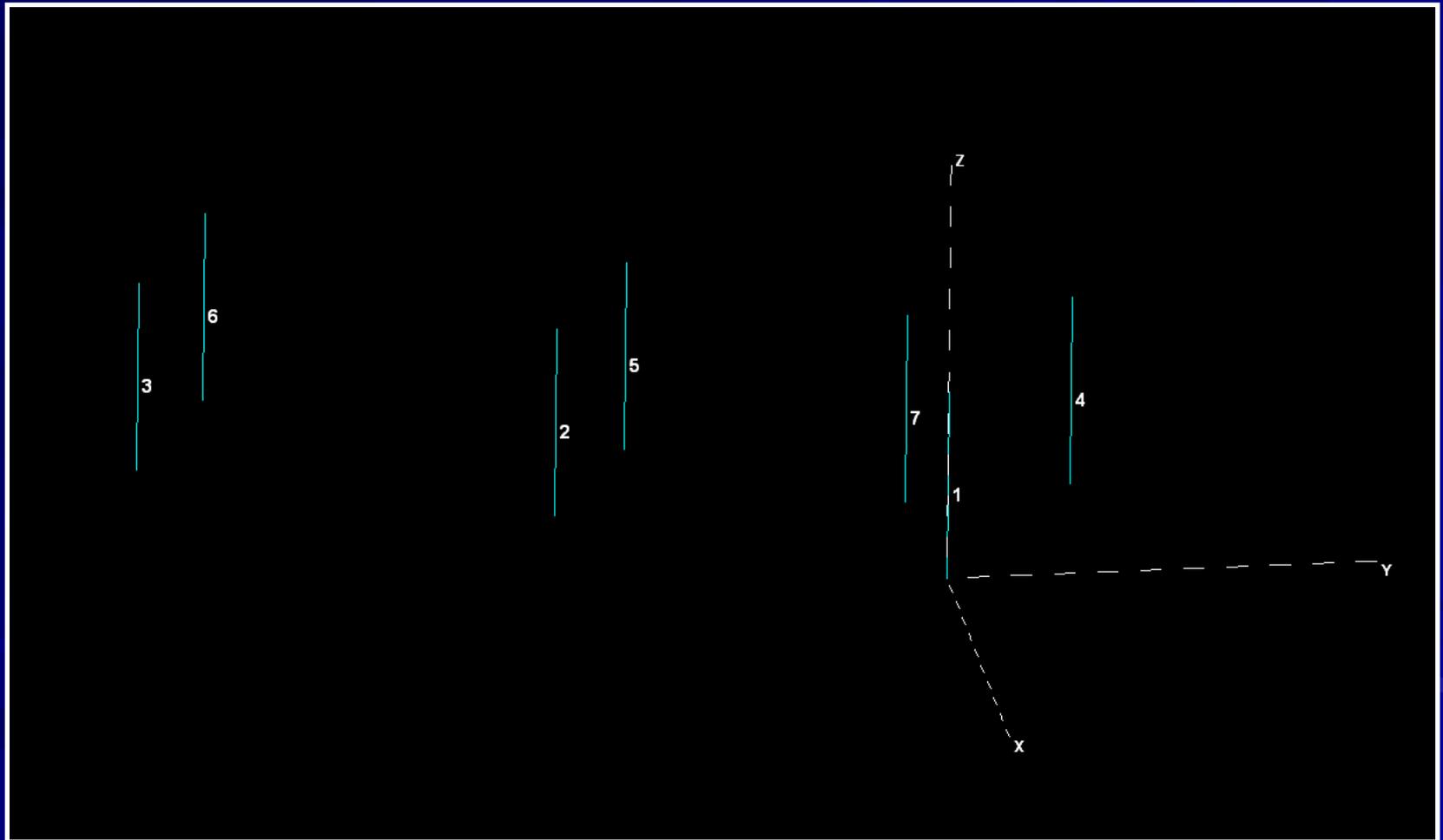
GEOMETRY

Wire coordinates in degrees; other dimensions in meters

Environment: perfect ground

wire	caps	Distance	Angle	Z	radius	segs
1	none	0	0	0	.29	10
		0	0	84.6		
2	none	181.2	120.2	0	.29	10
		181.2	120.2	84.6		
3	none	366.4	116.6	0	.29	10
		366.4	116.6	84.6		
4	none	123.8	213.6	0	.29	10
		123.8	213.6	84.6		
5	none	201.7	144.9	0	.29	10
		201.7	144.9	84.6		
6	none	375.6	130.1	0	.29	10
		375.6	130.1	84.6		
7	none	90.	176.7	0	.29	10
		90.	176.7	84.6		

Example



Example

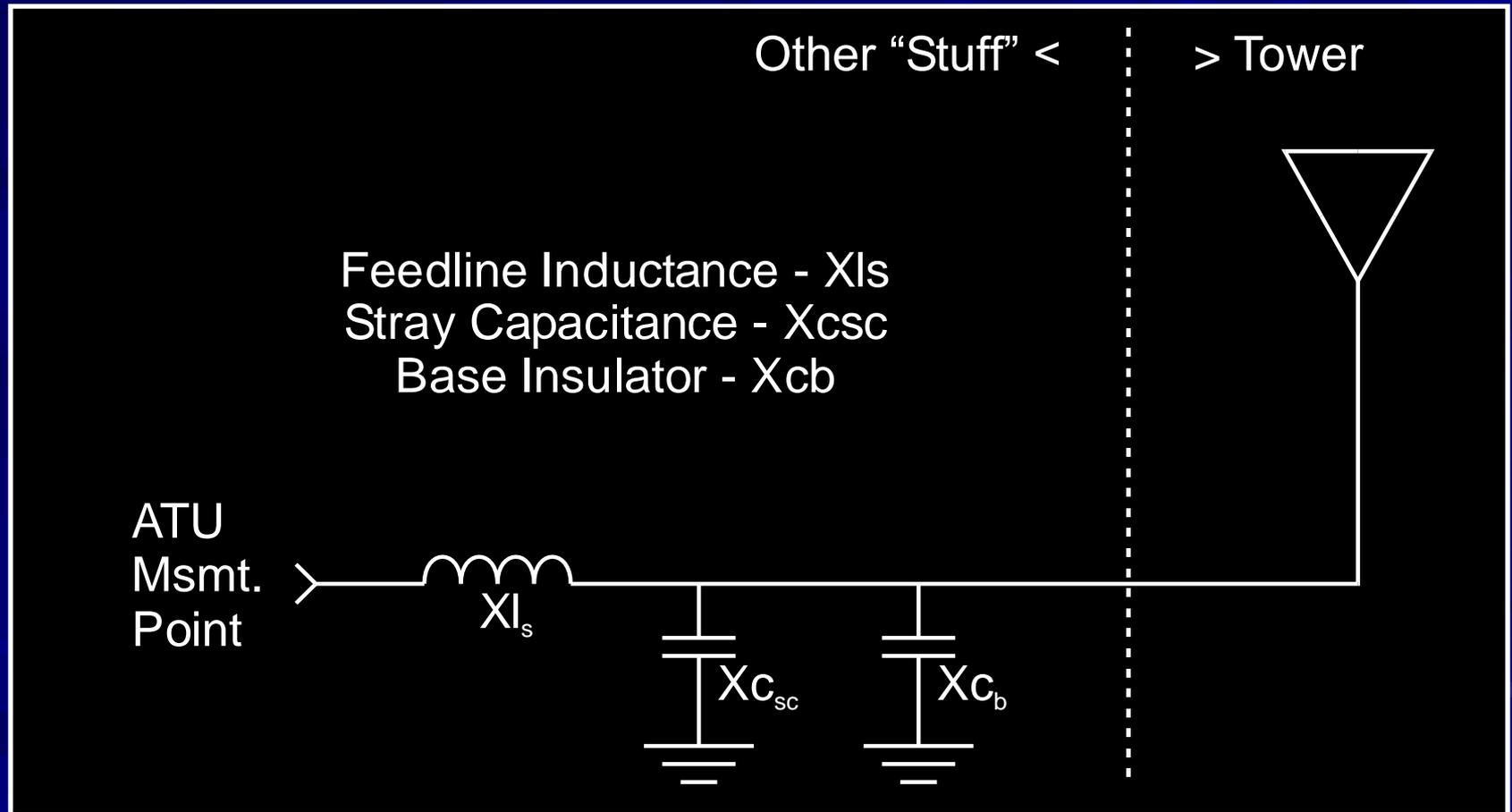
```
IMPEDANCE
normalization = 50.
freq      resist  react  imped  phase  VSWR  S11    S12
(KHz)    (ohms) (ohms) (ohms) (deg)
source = 1; node 1, sector 1
1,180.   33.353  -7.1002  34.101  348.   1.5521  -13.297  -.20817
```

Model Impedance for all towers: $33.4 -j7.1$ Ohms

Measured Impedance for tower 1: $39.1 +j60.2$ Ohms

Not Acceptable

Verifying the Model



Verifying the Model

■ With Series Inductance

$$Z_{atu} = R_b + jX_b + jX_{ls}$$

■ With Shunt Capacitance

$$R_{atu} = \frac{R_b * X_{cs}^2}{R_b^2 + (X_b + X_{cs})^2}$$
$$X_{atu} = \frac{jX_{cs} * (R_b^2 + X_b^2 + X_b * X_{cs})}{R_b^2 + (X_b + X_{cs})^2}$$

Verifying the Model

Change Velocity of Propagation

Slower velocity = taller tower

Measured Impedance for tower 1

39.1 +j60.2 Ohms

Model Impedance for V_p of 95.3% (105.2% height)

39.1 +j13.8 Ohms

Resistance is now Acceptable

Example

GEOMETRY

Wire coordinates in degrees; other dimensions in meters

Environment: perfect ground

wire	caps	Distance	Angle	Z	radius	segs
1	none	0	0	0	.29	10
		0	0	88.8		
2	none	181.2	120.2	0	.29	10
		181.2	120.2	88.5		
3	none	366.4	116.6	0	.29	10
		366.4	116.6	88.2		
4	none	123.8	213.6	0	.29	10
		123.8	213.6	88.		
5	none	201.7	144.9	0	.29	10
		201.7	144.9	88.6		
6	none	375.6	130.1	0	.29	10
		375.6	130.1	88.2		
7	none	90.	176.7	0	.29	10
		90.	176.7	90.2		

Example

Number of wires = 7
current nodes = 70

	minimum		maximum	
Individual wires	wire	value	wire	value
segment length	4	8.8	7	9.02
radius	1	.29	1	.29

ELECTRICAL DESCRIPTION

Frequencies (KHz)

frequency			no. of steps	segment length (wavelengths)	
no.	lowest	step		minimum	maximum
1	1,180.	0	1	.0244444	.0250556

Sources

source	node	sector	magnitude	phase	type
1	1	1	1.	0	voltage

Example

Lumped loads

load	node	resistance (ohms)	reactance (ohms)	inductance (mH)	capacitance (uF)	passive circuit
1	1	0	0	0	0	0
2	11	0	-10,000.	0	0	0
3	21	0	-10,000.	0	0	0
4	31	0	-10,000.	0	0	0
5	41	0	-10,000.	0	0	0
6	51	0	-10,000.	0	0	0
7	61	0	-10,000.	0	0	0

IMPEDANCE

normalization = 50.

freq (KHz)	resist (ohms)	react (ohms)	imped (ohms)	phase (deg)	VSWR	S11 dB	S12 dB
source = 1; node 1, sector 1							
1,180.	39.052	13.775	41.41	19.4	1.4853	-14.187	-.16884

Example

CURRENT rms

Frequency = 1180 KHz

Input power = .0113867 watts

Efficiency = 100. %

coordinates in degrees

current

no.	X	Y	Z	mag (amps)	phase (deg)	real (amps)	imaginary (amps)
GND	0	0	0	.0170758	340.6	.0161033	-5.68E-03
2	0	0	8.88	.0170977	338.6	.0159227	-6.23E-03
3	0	0	17.76	.0166592	337.4	.0153845	-6.39E-03
4	0	0	26.64	.0158121	336.5	.014499	-6.31E-03
5	0	0	35.52	.0145774	335.7	.0132834	-6.E-03
6	0	0	44.4	.012979	335.	.0117601	-5.49E-03
7	0	0	53.28	.011045	334.3	9.96E-03	-4.78E-03
8	0	0	62.16	8.8E-03	333.8	7.9E-03	-3.89E-03
9	0	0	71.04	6.28E-03	333.2	5.61E-03	-2.83E-03
10	0	0	79.92	3.46E-03	332.7	3.08E-03	-1.59E-03
END	0	0	88.8	0	0	0	0

Verifying the Model

- Adjust Tower Impedance Matrix
 - “Massage” model to agree with values measured at the ATU output
 - Adjust model height (velocity of propagation)
 - Add lumped feed inductance (series)
 - Compensate for base insulator capacitance
 - Repeat for each tower in array
 - An iterative process

FCC Requirements

- Only arrays with series-fed elements
- At least 1 segment per 10 degrees height
- Modeled radii between 80% and 150% of circle of circumference equal to sum of sides
- Modeled height between 75% and 125% of physical tower height

FCC Requirements

- Tapered towers may be modeled as a series of stepped cylinders (wedding cake)
- Actual tower spacings and orientation must be used in the model
 - This needs to be confirmed by a surveyor
- Lumped series inductance limited to 10 μH
- Lumped shunt capacitance limited to 250 pF

FCC Requirements

- Other base region components (lighting chokes, etc.) must be specifically measured and included in the model
- Measured impedance matrix must agree with moment method model within ± 2 ohms or $\pm 4\%$, whichever is greater

Example

Two turn lightning
loops placed in tower
feeds
(all 7 towers)

Loops measured
between
 $j33.9$ and $j46.6$ Ohms
($4.6 - 6.3$ μH)



Example

Tower	Msd	Msd	Msd Feed	Calc	Model	Model	Model	Model	Model	Model
	R	X	X	X	Height	% of 84.6	R	R % dev	X	X dev
1	39.1	60.2	40.3	19.9	88.8	105.2	39.1	-0.12	13.8	-6.15
2	39.8	61.0	44.5	16.5	88.5	104.9	39.8	-0.09	11.6	-4.89
3	38.7	50.0	33.9	16.1	88.2	104.5	38.6	-0.20	9.8	-6.26
4	37.6	45.8	34.8	11.0	88.0	104.3	37.6	-0.05	8.3	-2.67
5	39.7	60.2	46.6	13.6	88.6	105.0	39.6	-0.17	12.0	-1.56
6	38.7	43.2	38.1	5.1	88.2	104.5	38.7	0.03	10.0	4.92
7	39.0	58.5	38.6	19.9	90.2	106.9	39.0	-0.12	20.9	0.94

Model R's now within 0.1 Ohm of measured R's
 Model tower heights at 104% to 107% of actual
 Model lumped inductance less than 10 uH

Modeling

■ Pattern Shape Defined by Formula

$$E(\phi, \theta)_{theo} = \left| k \sum_{i=1}^n E_i * f_i(\theta) \angle (S_i \cos \theta \cos(\phi_i - \phi) + \phi_i) \right|$$

(FCC Rules 73.150)

■ The Sum of Individual Tower Fields

- Field ratio and phase for each tower
- Specified in construction permit

Modeling

- Measuring the Tower Field is Difficult
 - As with NDA, measure currents, however...

Establishing Tower Drive Impedances in Directionals is Very Difficult

- Impedance is really “Volts-per-Ampere”
 - Tower drive impedance is a summation
 - Self-impedance plus other “volts-per-ampere”

Modeling

- Credit to Jerry Westburg:
 - “Matrix Method for Relating Base Current Ratios to Field Ratios of AM Directionals”
 - IEEE Trans. On Bcstg. V 35 #2, June, 1989
- Calculate Drive Impedances and Currents
 - Verified NEC mathematical model
 - Field ratios from construction permit

Example

MEDIUM WAVE ARRAY SYNTHESIS FROM FIELD RATIOS

Frequency = 1180 KHz

	field ratio	
tower	magnitude	phase (deg)
1	1.	0
7	.894	109.2

VOLTAGES AND CURRENTS - rms

source voltage			current	
node	magnitude	phase (deg)	magnitude	phase (deg)
1	1,730.9	40.8	27.9853	5.2
61	419.553	128.1	26.2662	111.1

Sum of square of source currents = 2,948.56

Total power = 50,000. watts

Normalize phase ($111.1 - 5.2$), Tower 7 Phase = $+105.9^\circ$

Ratio current ($26.2662 / 27.9853$), Tower 7 Ratio = 0.939

Example

MEDIUM WAVE ARRAY SYNTHESIS FROM FIELD RATIOS

tower	field ratio magnitude	phase (deg)
1	.438	-6.9
2	1.	0
3	.523	16.7
4	.418	64.9
5	.99	99.5
6	.549	122.

VOLTAGES AND CURRENTS - rms

source	voltage magnitude	phase (deg)	current magnitude	phase (deg)
1	246.145	333.3	3.69944	.1
11	415.624	17.3	7.85442	5.5
21	184.362	23.4	4.18681	21.5
31	38.5047	339.6	3.44322	65.1
41	56.0734	45.7	8.02852	100.
51	13.2224	329.3	4.43341	121.8

Sum of square of source currents = 378.234

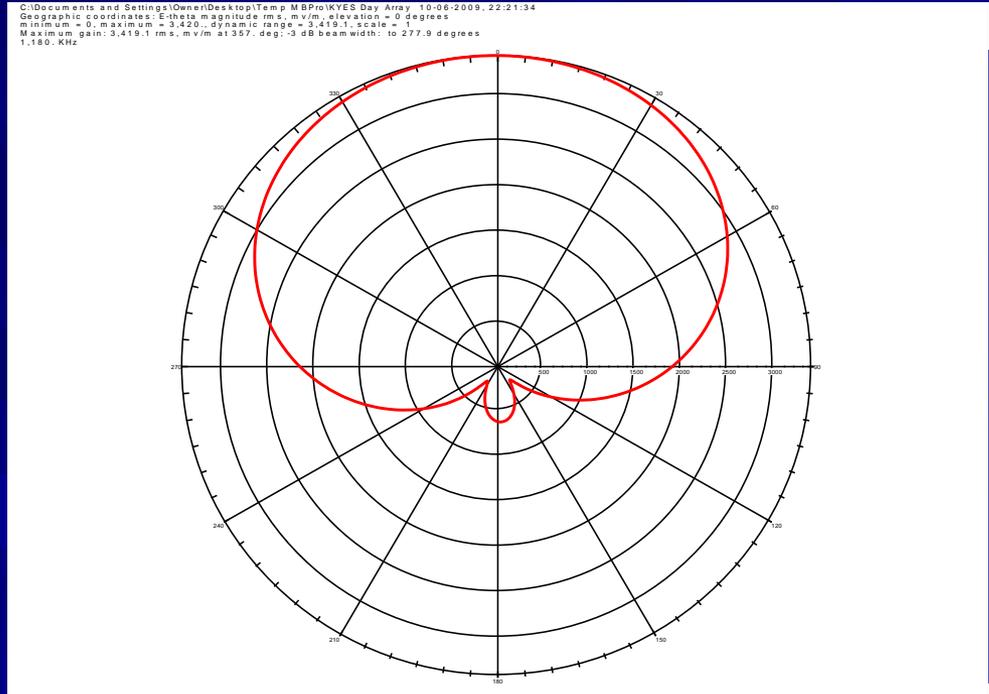
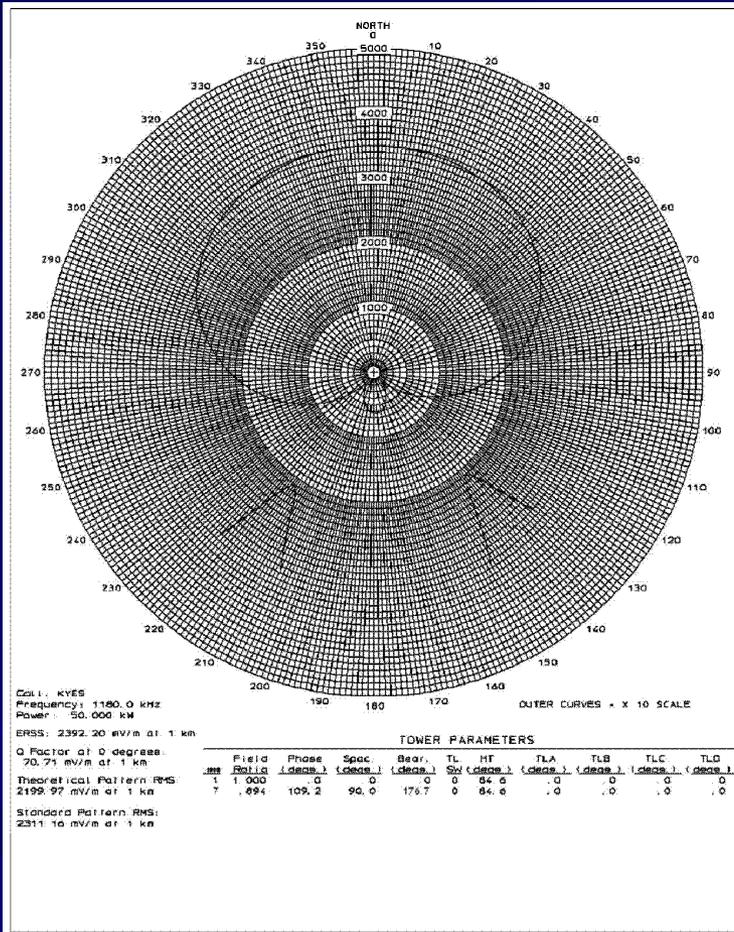
Total power = 5,000. watts

Example

Day Pattern - 50 KW.								
Tower	Theo.	Theo.	Model	Model	Model	Model	Ant. Mon.	Ant. Mon.
	Field	Phase	R	X	Current	Phase	Ratio	Phase
(Ref) 1	1	0	50.323	35.981	27.9982	5.2	1.000	0.0
7	0.894	109.2	15.269	4.6807	26.2882	111.1	0.939	105.9
Night Pattern - 5 KW.								
Tower	Theo.	Theo.	Model	Model	Model	Model	Ant. Mon.	Ant. Mon.
	Field	Phase	R	X	Current	Phase	Ratio	Phase
1	0.438	-6.9	59.424	-31.339	3.69731	0.1	0.471	-5.4
(Ref) 2	1	0	51.818	10.779	7.85258	5.5	1.000	0.0
3	0.523	16.7	44.009	1.4782	4.18673	21.5	0.533	16.0
4	0.418	64.9	0.8838	-11.147	3.44338	65.1	0.439	59.6
5	0.99	99.5	4.0698	-5.6732	8.03095	100	1.023	94.5
6	0.549	122	-2.6455	-1.3779	4.43277	121.8	0.564	116.3

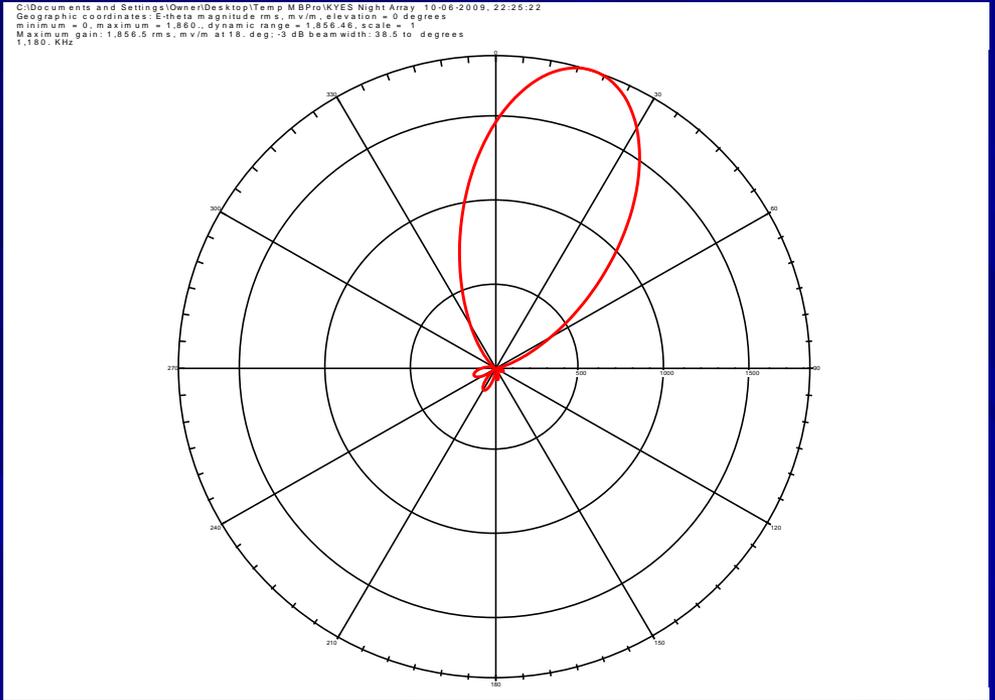
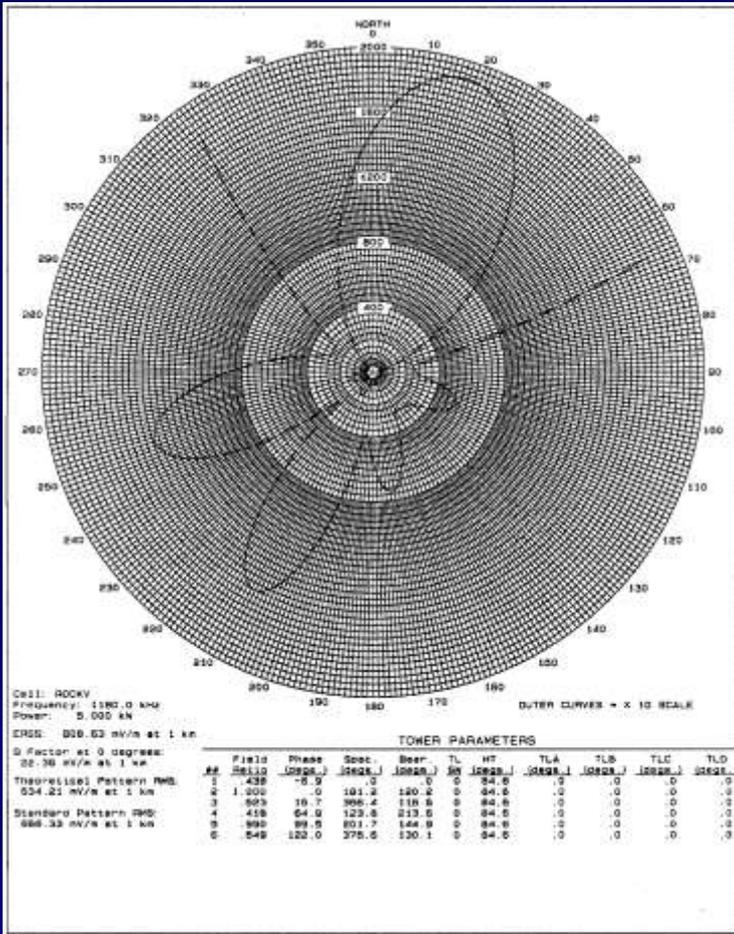
Operating Parameters Calculated!

Example



CP pattern versus
 NEC generated pattern

Example



CP pattern versus
 NEC generated pattern

DA FCC “302”

- Location – from Surveyor
- Pattern Shape – Method of Moments
- Pattern Size – calculated CP current

We have obtained most of the required technical information to prepare the license application without turning on the transmitter.

But, we still need to tune up the phasor...

Example

■ Tune-up:

- Day, 2 towers - about 4 hours
- Night, 6 towers - about 4 days
 - More fussing required at ATU's
 - Some very low tower drive impedances
- Measurements to characterize the array
- Time to “massage” the model

FCC Requirements

- Determining “Proper Adjustment”
 - Correlation of the Method of Moments parameters and antenna monitor indications
 - Ratio +/- 5%
 - Phase +/- 3 degrees
 - Correlation of Method of Moments and measured matrix impedances
 - Within +/- 2 ohms, +/- 4% for both resistance and reactance

Sampling System

■ “Classic” Proof

- Pattern adjusted by field measurements
- Antenna parameters are a “snapshot”
 - Values are a relative indication of operation

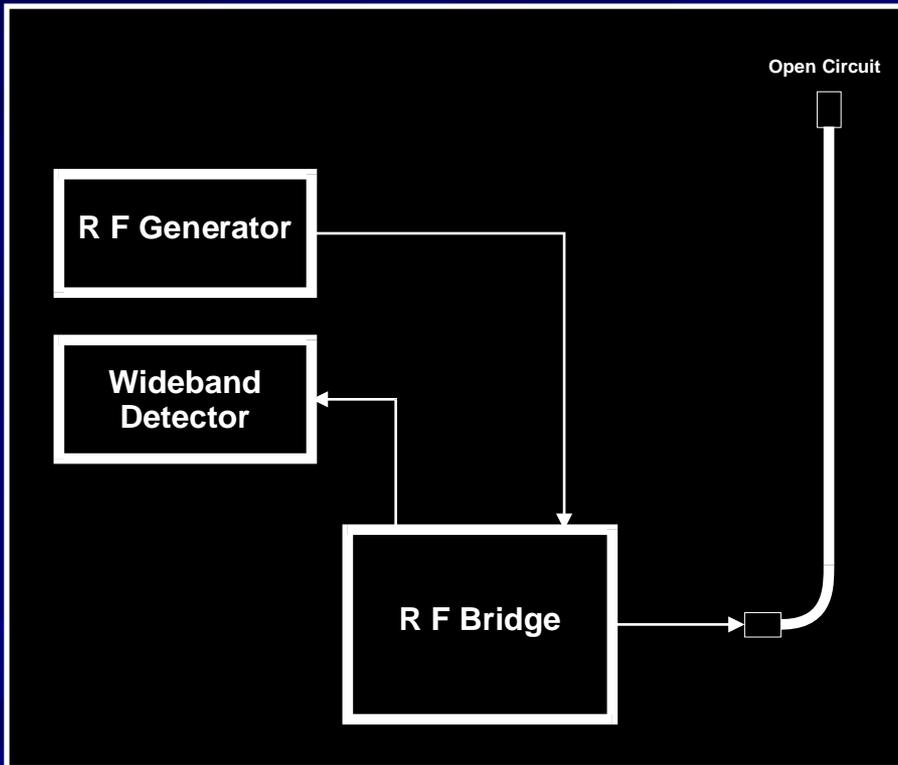
■ Method of Moments Proof

- Pattern adjusted by antenna parameters
 - Values are an absolute indication of operation

Sampling System

- The Sampling System Must be Accurate
 - Verified equal phase shift in all lines
 - Verified equal impedance for all lines
 - Verified equal sample toroid characteristics
 - Verified accurate antenna monitor
- System Measurements
 - Network analyzer or impedance bridge

Sample Line Verification



Line Open Circuit
Tuneable RF source
Oscilloscope
GR 916AL Bridge

Sample Lines

■ Verifying Length and Impedance

- Find nearest sample line resonant frequency
 - Open circuit line
 - Shorts occur at odd multiples of $\frac{1}{4}$ wavelength
 - Ratio frequencies for sample line length at carrier
- Measure lines at + and – $\frac{1}{8}$ wavelength

$$Z_0 = \sqrt{\left(\sqrt{R_{\frac{+\pi}{4}}^2 * X_{\frac{+\pi}{4}}^2} \right) * \left(\sqrt{R_{\frac{-\pi}{4}}^2 * X_{\frac{-\pi}{4}}^2} \right)}$$

Example

Sample line measurements with GR-916AL bridge

Null measurements made with far line end OPEN

	Null #1	Null #2	Null #3	1180 KHz	Offset	Offset
Tower	90 deg	270 deg	450 deg	Length	+45 deg	-45 deg
	Khz.	Khz.	Khz.	deg.	Khz.	Khz.
1	318.0	961.0	1609.3	331.5	1121.2	800.8
2	318.0	961.2	1609.6	331.5	1121.4	801.0
3	318.0	961.2	1609.5	331.5	1121.4	801.0
4	318.0	961.1	1609.6	331.5	1121.3	800.9
5	318.0	961.1	1609.5	331.5	1121.3	800.9
6	318.1	961.3	1609.9	331.4	1121.5	801.1
7	318.0	961.1	1609.6	331.5	1121.3	800.9

Example

Sample line measurements with GR-916AL bridge

Null measurements made with far line end OPEN;

	Meas.	Meas.	Meas.	Meas.	Calc.	Line and Toroid	
Tower	R (+45)	X (+45)	R (-45)	X (-45)	line Z	Meas. R	Meas. X
	ohms	ohms	ohms	ohms	ohms	ohms	ohms
1	5.6	48.1	4.4	-50.6	49.6	49.2	1.3
2	5.6	48.1	4.3	-50.6	49.6	48.6	1.7
3	5.7	48.1	4.3	-50.6	49.6	48.9	1.3
4	5.6	48.1	4.3	-50.6	49.6	48.7	1.3
5	5.7	48.1	4.3	-50.6	49.6	48.8	1.7
6	5.6	48.1	4.3	-50.6	49.6	48.6	1.3
7	5.7	48.1	4.4	-50.6	49.6	48.8	1.3

Sample System

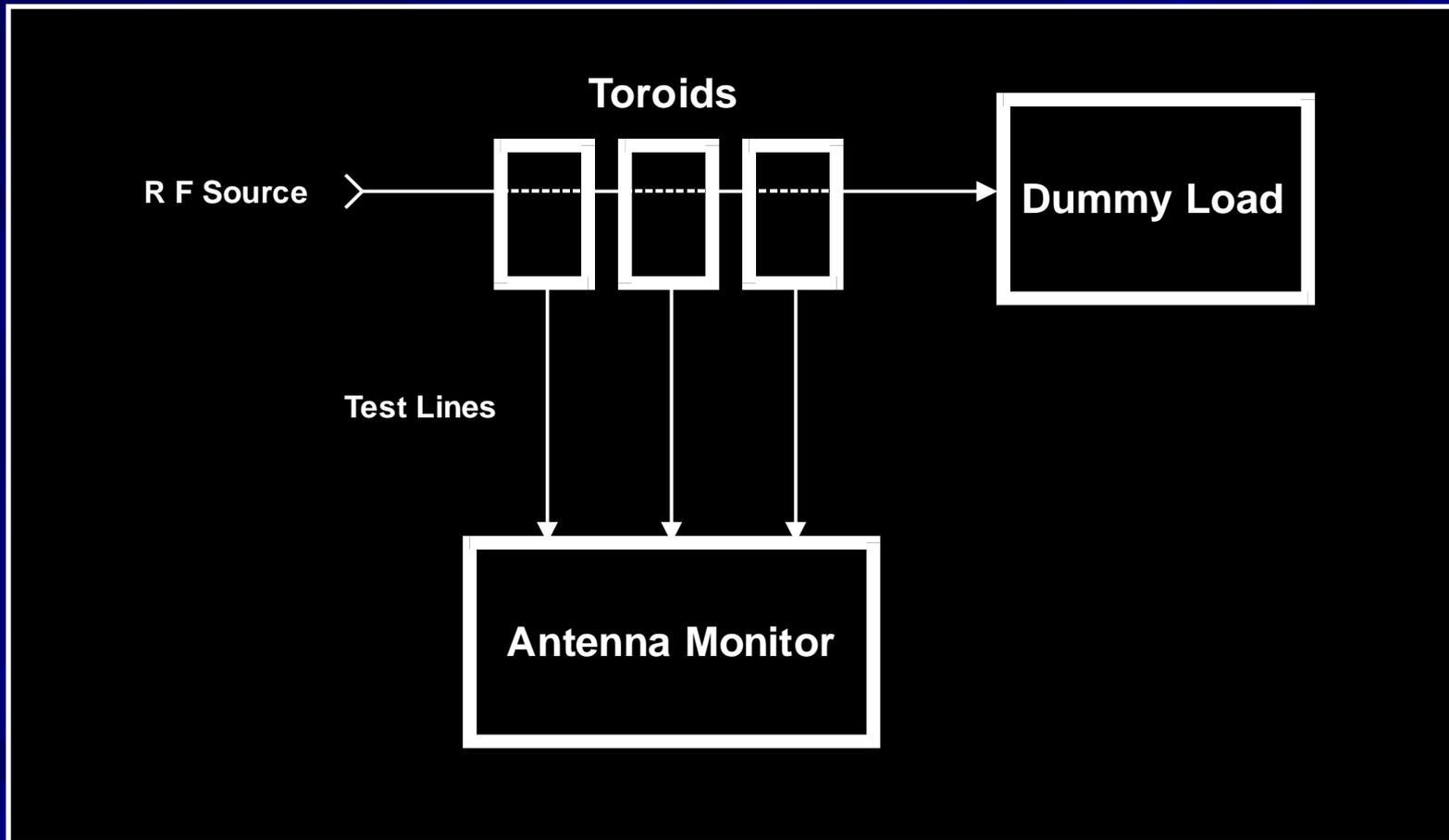
■ Verifying Toroids

- Build a test fixture
- Common RF current
- Toroids closely spaced
- Use antenna monitor for measurement

■ Verifying Sample Loops

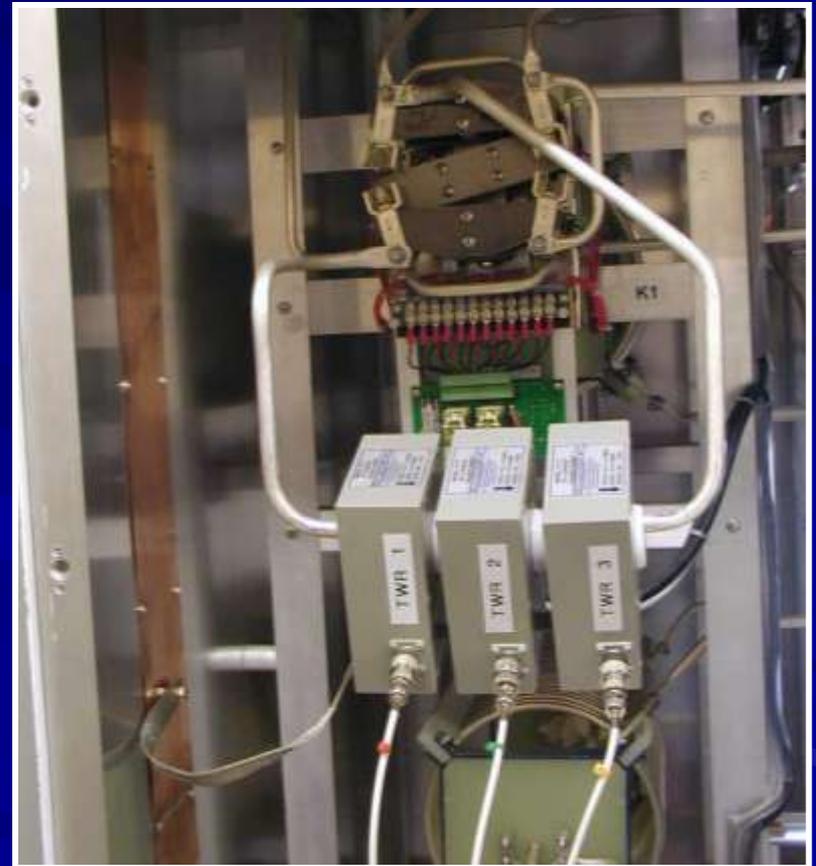
- More complicated
- FCC Rules 73.151, FCC 08-228

Toroid Verification



Example

- Toroid Test Fixture
 - Always ground toroids!
 - Always terminate!
- Operate into Load
- Used Potomac 1900
 - Equal Length cables
 - 1.5 RF Volts minimum



FCC Requirements

- Sample line lengths must agree within 1 electrical degree
- Sample line characteristic impedances must agree within 2 ohms
- Toroids allowed for tower heights less than 120 degrees or greater than 190 degrees

Field Measurements

- Some Field Measurements Still Required
 - On pattern minima and maxima
 - Three points per radial
 - Only reference measurements
- Not Monitor Points, No Regular Readings
 - However, these measurements (and some others) must be periodically repeated

Maintenance

- At Least Once Every 24 Months
 - Sampling system
 - Recertify toroid performance
 - Common reference signal calibration
 - Recertify sample line performance
 - Length and characteristic impedance
 - Repeat reference field measurements
 - Retain in Public Inspection File

The Decision

Method of Moments

or

“Classic” Proof